1a $P(\text{som}=6)=P(\underline{\underline{24}})+P(33)=\binom{2}{1}\cdot P(24)+P(33)=2\cdot\frac{2}{4}\cdot\frac{1}{4}+\frac{1}{4}\cdot\frac{1}{4}=\frac{4}{16}+\frac{1}{16}=\frac{5}{16}.$

1b $P(\text{som}=10)=P(\underline{\underline{334}})+P(\underline{\underline{244}})=\binom{3}{2}\cdot P(334)+\binom{3}{1}\cdot P(244)=3\cdot\frac{2}{6}\cdot\frac{2}{6}\cdot\frac{2}{6}+3\cdot\frac{1}{6}\cdot\frac{2}{6}\cdot\frac{2}{6}=\frac{24}{216}+\frac{12}{216}=\frac{36}{216}=\frac{1}{6}.$

1c $T=$ het aantal keer 2 $\Rightarrow P(T\geq 3)=1-P(T\leq 2)=1-\text{binomcdf}(12,\frac{2}{4},2)\approx 0{,}981.$

1d $P(\underline{\underline{24}})=P(24)+P(42)=\frac{2}{4}\cdot\frac{2}{6}+\frac{1}{4}\cdot\frac{1}{6}=\frac{4}{24}+\frac{1}{24}=\frac{5}{24}.$

1e $E=$ het aantal keer 1 $\Rightarrow P(E\geq 3)=1-P(E\leq 2)=1-\text{binomcdf}(n,\frac{1}{6},2)>0{,}85.$
$n=26$ (TABLE) $\Rightarrow P(E\geq 3)\approx 0{,}832$ en $n=27$ (TABLE) $\Rightarrow P(E\geq 3)\approx 0{,}851.$
Je moet dus minstens 27 keer draaien.

1f $P(\underline{\underline{222334}})=\binom{6}{3}\cdot\binom{3}{2}\cdot\left(\frac{2}{4}\right)^3\cdot\left(\frac{1}{4}\right)^2\cdot\frac{1}{4}\approx 0{,}117$ of $\frac{6!}{3!\cdot 2!}\cdot\left(\frac{2}{4}\right)^3\cdot\left(\frac{1}{4}\right)^2\cdot\frac{1}{4}\approx 0{,}117.$

1g $P(**\bar{4}444)=1\cdot 1\cdot\frac{4}{6}\cdot\left(\frac{2}{6}\right)^3\approx 0{,}025.$ (waarbij * staat voor "alles mag" en $\bar{4}$ voor "geen 4")

2a $P(\underline{\underline{rrr\bar{r}\bar{r}}})=\frac{\binom{7}{3}\cdot\binom{8}{2}}{\binom{15}{5}}\approx 0{,}326$ of $\binom{5}{3}\cdot\frac{7}{15}\cdot\frac{6}{14}\cdot\frac{5}{13}\cdot\frac{8}{12}\cdot\frac{7}{11}\approx 0{,}326.$

2b $P(\underline{\underline{rrrr\bar{r}\bar{r}\bar{r}\bar{r}}})=\binom{8}{4}\cdot\left(\frac{7}{15}\right)^4\cdot\left(\frac{8}{15}\right)^4\approx 0{,}269$ of $\text{binompdf}(8,\frac{7}{15},4)\approx 0{,}269.$

2c $P(\underline{\underline{rrrwwz}})=\frac{\binom{7}{3}\cdot\binom{5}{2}\cdot\binom{3}{1}}{\binom{15}{6}}\approx 0{,}210$ of $\binom{6}{3}\cdot\binom{3}{2}\cdot\frac{7}{15}\cdot\frac{6}{14}\cdot\frac{5}{13}\cdot\frac{5}{12}\cdot\frac{4}{11}\cdot\frac{3}{10}\approx 0{,}210.$

2d $P(\underline{\underline{rrrwwz}})=\binom{6}{3}\cdot\binom{3}{2}\cdot\left(\frac{7}{15}\right)^3\cdot\left(\frac{5}{15}\right)^2\cdot\frac{3}{15}\approx 0{,}136.$

2e $P(\bar{r}\bar{r}\bar{r}\bar{r}r)=\frac{8}{15}\cdot\frac{7}{14}\cdot\frac{6}{13}\cdot\frac{5}{12}\cdot\frac{7}{11}\approx 0{,}033$ of $P(\bar{r}\bar{r}\bar{r}\bar{r}r)=P(\bar{r}\bar{r}\bar{r}\bar{r})\cdot P(r)=\frac{\binom{8}{4}}{\binom{15}{4}}\cdot\frac{7}{11}\approx 0{,}033.$

2f $P(\underline{\underline{rr\bar{r}\bar{r}\bar{r}\bar{r}}}r)=\binom{6}{2}\cdot\frac{7}{15}\cdot\frac{6}{14}\cdot\frac{8}{13}\cdot\frac{7}{12}\cdot\frac{6}{11}\cdot\frac{5}{10}\cdot\frac{5}{9}\approx 0{,}163$ of $P(\underline{\underline{rr\bar{r}\bar{r}\bar{r}\bar{r}}}r)=P(\underline{\underline{rr\bar{r}\bar{r}\bar{r}\bar{r}}})\cdot P(r)=\frac{\binom{7}{2}\cdot\binom{8}{4}}{\binom{15}{6}}\cdot\frac{5}{9}\approx 0{,}163.$

3a $P(zz)=\frac{\binom{6}{2}}{\binom{20}{2}}=\frac{3}{38}\approx 0{,}079$ of $\frac{6}{20}\cdot\frac{5}{19}=\frac{3}{38}\approx 0{,}079.$
$X=$ het aantal keer "2 zwart" $\Rightarrow P(X=3)=\text{binompdf}(10,\text{Ans},3)\approx 0{,}033.$

3b $P(\text{twee met dezelfde kleur})=P(ww)+P(zz)+P(bb)=\frac{\binom{5}{2}}{\binom{20}{2}}+\frac{\binom{6}{2}}{\binom{20}{2}}+\frac{\binom{9}{2}}{\binom{20}{2}}\approx 0{,}321$ of
$P(\text{twee met dezelfde kleur})=P(ww)+P(zz)+P(bb)=\frac{5}{20}\cdot\frac{4}{19}+\frac{6}{20}\cdot\frac{5}{19}+\frac{9}{20}\cdot\frac{8}{19}=\frac{61}{190}\approx 0{,}321.$
$Y=$ het aantal keer "2 met dezelfde kleur" $\Rightarrow P(Y>4)=1-P(Y\leq 4)=1-\text{binomcdf}(10,\text{Ans},4)\approx 0{,}189.$

3c $P(\text{hoogstens 1 blauwe})=1-P(bb)=1-\frac{\binom{9}{2}}{\binom{20}{2}}\approx 0{,}811$ of $1-\frac{9}{20}\cdot\frac{8}{19}=\frac{77}{95}\approx 0{,}811.$
$B=$ het aantal keer "hoogstens één blauwe" $\Rightarrow P(B\geq 5)=1-P(B\leq 4)=1-\text{binomcdf}(10,\text{Ans},4)\approx 0{,}995.$

3d $P(s)=P(ww)=\frac{\binom{5}{2}}{\binom{20}{2}}\approx 0{,}053$ of $\frac{5}{20}\cdot\frac{4}{19}=\frac{1}{19}\approx 0{,}053.$ $P(\bar{s}\bar{s}\bar{s}s)=\left(\frac{18}{19}\right)^3\cdot\frac{1}{19}\approx 0{,}045.$

3e $P(\text{meer dan 3 keer pakken})=P(\bar{s}\bar{s}\bar{s})=\left(\frac{18}{19}\right)^3\approx 0{,}850.$

4a $P(R\geq 2)=1-P(T\leq 1)=1-\text{binomcdf}(5,0.80,1)\approx 0{,}993.$

4b $P(\text{afwisselend raak en mis})=P(rmrmr)+P(mrmrm)=0{,}8^3\cdot 0{,}2^2+0{,}2^3\cdot 0{,}8^2\approx 0{,}026.$

4c $P(\text{twee keer achter elkaar raak en drie keer mis})=P(\underline{\underline{\boxed{rr}\,mmm}})=\binom{4}{1}\cdot 0{,}8^2\cdot 0{,}2^3\approx 0{,}020.$

4d $P(\text{twee keer achter elkaar raak en drie keer mis}) = P(\underline{\underline{rmm}}\ \underline{rm}) = \binom{3}{1}\cdot 0{,}8\cdot 0{,}2^2\cdot\binom{2}{1}\cdot 0{,}8\cdot 0{,}2 \approx 0{,}031.$

```
3 nCr 1*2 nCr 1*
0.8²*0.2^3
            .03072
```

4e M = het aantal keer mis $\Rightarrow P(M \le 2) = \text{binomcdf}(10, 0.15, 2) \approx 0{,}820.$

```
binomcdf(10,0.15
,2)
      .8201964804
```

5a $P(\text{Amerikanen in de middelste drie banen}) = \dfrac{\binom{3}{3}}{\binom{7}{3}} \approx 0{,}029.$

```
3 nCr 3/7 nCr 3
      .0285714286
```

5b $P(\text{één Duitser in een buitenbaan}) = \dfrac{\binom{2}{1}\cdot\binom{5}{1}}{\binom{7}{2}} \approx 0{,}476.$

```
2 nCr 1*5 nCr 1/
7 nCr 2
      .4761904762
```

5c $P(\text{tenminste één niet-Amerikaan in een buitenbaan}) = 1 - P(\text{geen niet-Amerikaan in een buitenbaan}) = 1 - \dfrac{\binom{3}{2}}{\binom{7}{2}} \approx 0{,}857.$

```
1-3 nCr 2/7 nCr
2
      .8571428571
```

6 $P(1\ 2\ 3) = \frac{1}{10}\cdot\frac{1}{9}\cdot\frac{1}{8} = \frac{1}{720}.$

```
10*9*8
               720
```

7a $P(\text{baantje}) = P(B) = 0{,}60 \Rightarrow P(\text{geen baantje}) = P(\bar{B}) = 0{,}40;$
$P(\text{baantje van meer dan 12 uur/week}) = P(M) = \frac{3}{4}\times 0{,}60 = 0{,}45 \Rightarrow P(\text{kleinere baan}) = P(K) = \frac{1}{4}\times 0{,}60 = 0{,}15$
$P(\underline{\underline{\bar{B}\bar{B}\bar{B}MMMMMMMMMMMMKKKKK}}) = \binom{20}{3}\cdot\binom{17}{12}\cdot 0{,}40^3\cdot 0{,}45^{12}\cdot 0{,}15^5 \approx 0{,}002.$

```
20 nCr 3*17 nCr
12*0.4^3*0.45^12
*0.15^5
       .002363968
```

7b $P(\text{minstens vijf keer bellen}) = P(\text{eerste vier keer geen succes}) = P(\bar{K}\bar{K}\bar{K}\bar{K}) = 0{,}85^4 \approx 0{,}522.$

```
0.85^4
        .52200625
```

7c 28 leerlingen, waarvan 16 een baantje. 5 van deze 16 werken meer dan 12 uur $\Rightarrow$ 11 hebben een kleinere baan
$P(\text{vier keer bellen}) = P(\bar{K}\bar{K}\bar{K}K) = \dfrac{\binom{17}{3}}{\binom{28}{3}}\cdot\frac{11}{25} \approx 0{,}091$ of $P(\bar{K}\bar{K}\bar{K}K) = \frac{17}{28}\cdot\frac{16}{27}\cdot\frac{15}{26}\cdot\frac{11}{25} \approx 0{,}091.$

```
17 nCr 3/28 nCr
3*11/25
      .0913308913
17*16*15*11/28/2
7/26/25
      .0913308913
```

8a $P(\underline{\underline{1\,1\,1\,1\,2\,2\,2\,2\,3\,3\,3\,3\,4\,4\,4\,4}}) = \binom{16}{4}\cdot\binom{12}{4}\cdot\binom{8}{4}\cdot\left(\frac{1}{4}\right)^{16} \approx 0{,}015$ of $\frac{16!}{4!\cdot 4!\cdot 4!\cdot 4!}\cdot\left(\frac{1}{4}\right)^{16} \approx 0{,}015.$

```
16 nCr 4*12 nCr
4*8 nCr 4*(1/4)^
16
      .0146829989
16!/(4!*4!*4!*4!
)*(1/4)^16
      .0146829989
```

8b $P(\underline{\underline{2\,2\,2\,2\,2\,2\,3\,3\,3\,3\,a\,a\,a\,a\,a\,a}}) = \binom{16}{6}\cdot\binom{10}{4}\cdot\left(\frac{1}{4}\right)^{10}\cdot\left(\frac{2}{4}\right)^{6} \approx 0{,}025$ of $\frac{16!}{6!\cdot 4!\cdot 6!}\cdot\left(\frac{1}{4}\right)^{10}\cdot\left(\frac{2}{4}\right)^{6} \approx 0{,}025.$
(a staat voor iets anders dan een 2 of een 3)

```
16 nCr 6*10 nCr
4*(1/4)^10*(2/4)
^6
      .0250589848
16!/(6!*4!*6!)*(
1/4)^10*(2/4)^6
      .0250589848
```

8c $P(\text{bij de tiende worp evenveel als bij de derde worp}) = \frac{1}{4} = 0{,}25.$

9a $P(\text{som is 5}) = P(5) = \frac{4}{16} = \frac{1}{4}$ (zie de tabel hiernaast).
$P(\underline{\underline{55\bar{5}\bar{5}}}) = \binom{4}{2}\cdot\left(\frac{1}{4}\right)^2\cdot\left(\frac{3}{4}\right)^2 = \frac{27}{128}$ of $\text{binompdf}(4, \frac{1}{4}, 2) = \frac{27}{128}.$

```
4 nCr 2*(1/4)²*(
3/4)²▸Frac
          27/128
binompdf(4,1/4,2
)▸Frac
          27/128
```

+	1	2	3	4
4	5	6	7	8
3	4	5	6	7
2	3	4	5	6
1	2	3	4	5

9b $P(\text{verschil is 2}) = P(2) = \frac{4}{16} = \frac{1}{4}$ (zie de tabel hiernaast). A = aantal keer "verschil is 2".
$P(A \ge 1) = 1 - P(A = 0) = 1 - \left(\frac{3}{4}\right)^4 = \frac{175}{256}$ of $1 - \text{binompdf}(4, \frac{1}{4}, 0) = \frac{175}{256}.$

```
1-(3/4)^4▸Frac
          175/256
1-binompdf(4,1/4
,0)▸Frac
          175/256
```

−	1	2	3	4
4	3	2	1	0
3	2	1	0	1
2	1	0	1	2
1	0	1	2	3

vaas	I	II
rood	6	a
zwart	$a-6$	$24-a$
totaal	a	24

10a $P(rr) = \frac{6}{a}\cdot\frac{a}{24} = \frac{6\cancel{a}}{24\cancel{a}} = \frac{1}{4}.$

10b $P(\underline{\underline{rz}}) = P(rz) + P(zr) = \frac{6}{a}\cdot\frac{24-a}{24} + \frac{a-6}{a}\cdot\frac{a}{24} = \frac{144-6a}{24a} + \frac{a^2-6a}{24a} = \frac{a^2-12a+144}{24a}.$

10c $P(\underline{\underline{rz}}) = \binom{2}{1}\cdot P(rz) = 2\cdot\frac{6}{a}\cdot\frac{a-6}{a-1} = \frac{2\cdot 6\cdot(a-6)}{a\cdot(a-1)} = \frac{12a-72}{a^2-a}.$

```
2 nCr 1
                 2
```

10d $P(\underline{\underline{rz}}) = \frac{12a-72}{a^2-a} > 0{,}4$ (bladeren door TABLE geeft)
8, 9, 10, 11, ... , 20, 21, 22 of 23 knikkers.

```
Plot1 Plot2 Plot3
\Y1■(12X-72)/(X²
-X)
\Y2■0.4
\Y3=
```

X	Y1	Y2
4	-2	.4
5	-.6	.4
6	0	.4
7	.28571	.4
8	.42857	.4
9	.5	.4
10	.53333	.4

X=10

X	Y1	Y2
19	.45614	.4
20	.44211	.4
21	.42857	.4
22	.41558	.4
23	.40316	.4
24	.3913	.4
25	.38	.4

X=25

11a E = het aantal dat met een 1 begint. (20% van 125 is $\frac{125}{5} = 25$)
$P(E < 25) = P(E \le 24) = \text{binomcdf}(125, 0.301, 24) \approx 0{,}004.$

```
binomcdf(125,30.
1/100,24)
      .0040152994
```

11b N = het aantal dat met een 9 begint. (10% van 80 is $\frac{80}{10} = 8$)
$P(N \ge 8) = 1 - P(N \le 7) = 1 - \text{binomcdf}(80, 0.046, 7) \approx 0{,}031.$

```
1-binomcdf(80,4.
6/100,7)
      .0307996572
```

11c Je verwacht dat (30,1% dus) $0{,}301\cdot 750 \approx 226$ aantallen met een 1 begint.
$P(E \le 189) = \text{binomcdf}(750, 0.301, 189) \approx 0{,}002.$
Er is aanleiding om aan fraude te denken, want de kans dat er 189 of minder met een 1 beginnen is heel erg klein.

```
binomcdf(750,30.
1/100,189)
      .0017018139
```

11d $P(\underline{\underline{1\,1\,1\,1\,2\,2\,2\,a\,a\,a\,a\,a}}) = \binom{12}{4}\cdot\binom{8}{3}\cdot 0{,}301^4\cdot 0{,}176^3\cdot 0{,}523^5 \approx 0{,}049$ of $\frac{12!}{4!\cdot 3!\cdot 5!}\cdot 0{,}301^4\cdot 0{,}176^3\cdot 0{,}523^5 \approx 0{,}049.$
(a staat voor een 3, 4, 5, 6, 7, 8 of 9)

```
30.1+17.6
             47.7
100-Ans
             52.3
```

```
12 nCr 4*8 nCr 3
*0.301^4*0.176^3
*0.523^5
      .0485406577
```

```
12!/4!/3!/5!*0.3
01^4*0.176^3*0.5
23^5
      .0485406577
```

12a Op de bovenste stippeltjes $P(\text{Sander pakt rood}) = P(r_S) = \frac{2}{4} = \frac{1}{2}$. Dus $\frac{1}{2}$.
Op de middelste stippeltjes $P(\text{Rob pakt wit}) = P(w_R) = \frac{2}{5}$. Dus $\frac{2}{5}$.
Op de onderste stippeltjes $P(\text{Sander pakt rood}) = P(r_S) = \frac{1}{4}$. Dus $\frac{1}{4}$.

12b $P(\text{Sander wint in 3 beurten}) = P(r_S\, w_R\, r_S) = \frac{2}{4} \cdot \frac{2}{5} \cdot \frac{1}{4} = \frac{1}{20} = 0{,}05.$

12c Het schema staat in figuur 13.3.

13a $P(\text{Anouk wint in eerste beurt}) = P(\text{Anouk pakt 3 keer rood}) = P(r_A\, r_A\, r_A) = \frac{5}{8} \cdot \frac{4}{7} \cdot \frac{3}{6} \approx 0{,}179.$

```
5*4*3/8/7/6
          .1785714286
```

13b $P(\text{Hinke wint bij het pakken van de vierde knikker}) = P(w_A\, r_H\, r_H\, r_H) = \frac{3}{8} \cdot \frac{5}{7} \cdot \frac{4}{6} \cdot \frac{3}{5} \approx 0{,}107.$

```
3*5*4*3/8/7/6/5
          .1071428571
```

13c $P(\text{Anouk wint bij het pakken van de vijfde knikker}) = P(w_A\, w_H\, r_A\, r_A\, r_A) + P(r_A\, w_A\, w_H\, r_A\, r_A) + P(r_A\, r_A\, w_A\, w_H\, r_A)$
$= \frac{3}{8} \cdot \frac{2}{7} \cdot \frac{5}{6} \cdot \frac{4}{5} \cdot \frac{3}{4} + \frac{5}{8} \cdot \frac{3}{7} \cdot \frac{2}{6} \cdot \frac{4}{5} \cdot \frac{3}{4} + \frac{5}{8} \cdot \frac{4}{7} \cdot \frac{3}{6} \cdot \frac{2}{5} \cdot \frac{3}{4} \approx 0{,}161.$

```
3*2*5*4*3+5*3*2*
4*3+5*4*3*2*3
                1080
Ans/(8*7*6*5*4)
          .1607142857
```

14a $P(\text{rrr}) = \frac{5}{8} \cdot \frac{7}{10} \cdot \frac{9}{12} \approx 0{,}328.$

```
5*7*9/8/10/12
             .328125
```

14b $P(\underline{\underline{\text{wwr}}}) = P(\text{wwr}) + P(\text{wrw}) + P(\text{rww}) = \frac{3}{8} \cdot \frac{5}{10} \cdot \frac{5}{12} + \frac{3}{8} \cdot \frac{5}{10} \cdot \frac{5}{12} + \frac{5}{8} \cdot \frac{3}{10} \cdot \frac{5}{12} \approx 0{,}234.$

```
(3*5*5+3*5*5+5*3
*5)/8/10/12
             .234375
3 nCr 2*3*5*5/8/
10/12
             .234375
```

15a $P(\text{EEEEE}) = 0{,}6^5 \approx 0{,}078.$

```
0.6^5
              .07776
```

15b $P(\underline{\underline{\text{EEBBBB}}}\text{B}) = \binom{6}{2} \cdot 0{,}6^2 \cdot 0{,}4^4 \cdot 0{,}4 \approx 0{,}055.$

```
6 nCr 2*0.6²*0.4
^5
             .055296
```

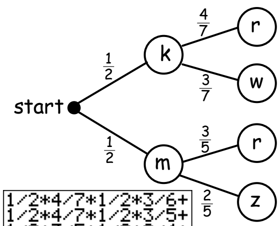
15c Dit betekent dat Eline de volgende drie rondes <u>moet</u> winnen met $P(\text{Eline wint alsnog}) = P(\text{EEE}) = 0{,}6^3 = 0{,}216.$

```
0.6^3
                .216
```

15d $P(\text{Eline wint alsnog}) = P(\text{EEE}) + P(\underline{\underline{\text{BEE}}}\text{E}) = 0{,}6^3 + \binom{3}{1} \cdot 0{,}4 \cdot 0{,}6^2 \cdot 0{,}6 \approx 0{,}475.$

```
0.6^3+3 nCr 1*0.
4*0.6^3
               .4752
```

16a Zie de kansboom (met de kansen) hiernaast.



16b $P(\text{Anton pakt zwart}) = P(\text{mz}) = \frac{1}{2} \cdot \frac{2}{5} = 0{,}2.$

16c $P(\text{Anton pakt rood}) = P(\text{kr}) + P(\text{mr}) = \frac{1}{2} \cdot \frac{4}{7} + \frac{1}{2} \cdot \frac{3}{5} \approx 0{,}586.$

```
1/2*4/7+1/2*3/5
          .5857142857
```

16d $P(\text{Anton pakt twee keer wit}) = P(\text{kwkw}) = \frac{1}{2} \cdot \frac{3}{7} \cdot \frac{1}{2} \cdot \frac{2}{6} \approx 0{,}036.$

```
1/2*3/7*1/2*2/6
          .0357142857
```

16e $P(\text{Anton pakt twee keer rood}) = P(\text{krkr}) + P(\text{krmr}) + P(\text{mrmr}) + P(\text{mrkr})$
$= \frac{1}{2} \cdot \frac{4}{7} \cdot \frac{1}{2} \cdot \frac{3}{6} + \frac{1}{2} \cdot \frac{4}{7} \cdot \frac{1}{2} \cdot \frac{3}{5} + \frac{1}{2} \cdot \frac{3}{5} \cdot \frac{1}{2} \cdot \frac{2}{4} + \frac{1}{2} \cdot \frac{3}{5} \cdot \frac{1}{2} \cdot \frac{4}{7} \approx 0{,}318.$

```
1/2*4/7*1/2*3/6+
1/2*4/7*1/2*3/5+
1/2*3/5*1/2*2/4+
1/2*3/5*1/2*4/7
          .3178571429
```

17a $P(\text{Evelien pakt de eerste keer rood}) = P(\text{Ir}) + P(\text{IIr}) = \frac{2}{6} \cdot \frac{4}{7} + \frac{4}{6} \cdot \frac{3}{5} \approx 0{,}590.$

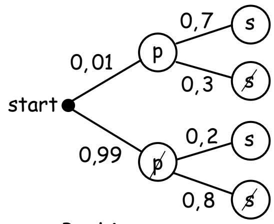
```
2/6*4/7+4/6*3/5
          .5904761905
Ans^3
          .2058766872
```

17b $P(\text{Evelien pakt drie keer rood}) = P(\text{rrr}) = \left(\frac{2}{6} \cdot \frac{4}{7} + \frac{4}{6} \cdot \frac{3}{5}\right)^3 \approx 0{,}206.$

17c $P(\text{Evelien pakt twee keer zwart}) = P(\text{IIzIIz}) = \frac{4}{6} \cdot \frac{2}{5} \cdot \frac{4}{6} \cdot \frac{2}{5} \approx 0{,}071.$

```
4/6*2/5
          .2666666667
Ans²
          .0711111111
```

18a Zie de kansboom hiernaast.



18b $P(\text{Nederlander heeft spierpijnklachten}) = P(\text{ps}) + P(\bar{\text{p}}\text{s}) = 0{,}01 \cdot 0{,}7 + 0{,}99 \cdot 0{,}2 = 0{,}205.$

```
0.01*0.7+0.99*0.
2
                .205
```

18c $\text{Aantal} = 10\,000 \cdot 0{,}01 \cdot 0{,}7 = 70.$

18d $\text{Aantal} = 10\,000 \cdot 0{,}205 = 2\,050.$

```
10000*0.01*0.7
                  70
10000*0.205
                2050
```

18e $P(\text{persoon met spierpijnklachten heeft Parkinson}) = \frac{70}{2050} \approx 0{,}034.$

```
70/2050
          .0341463415
```

18f Van de personen die spierpijnklachten hebben, heeft maar een klein deel de ziekte van Parkinson. (zie 18e)

19a $P(\text{testresultaat negatief}) = P(\text{geen malaria en negatief}) + P(\text{malaria en negatief}) = 0{,}94 \cdot 0{,}95 + 0{,}06 \cdot 0{,}20 = 0{,}905.$

```
0.94*0.95+0.06*0
.20
                .905
```

19b $P(\text{testresultaat positief}) = 1 - 0{,}905 = 0{,}095.$ Dus $P(\text{Marc is besmet}) = \frac{0{,}06 \cdot 0{,}80}{0{,}095} \approx 0{,}505.$

19c $P(\text{Sabine is niet besmet}) = \frac{0{,}94 \cdot 0{,}95}{0{,}905} \approx 0{,}987.$

```
1-0.905
                .095
0.06*0.8/0.095
          .5052631579
0.94*0.95/0.905
          .9867403315
```

20a $P(\text{Schut is aan het kopiëren}) = \frac{8}{60} = \frac{2}{15}.$

```
8/60▸Frac
                2/15
```

20b $P(\text{minstens één keer aan het kopiëren}) = 1 - P(\text{geen enkele keer aan het kopiëren}) = 1 - \left(\frac{52}{60}\right)^{10} \approx 0{,}761.$

```
1-(1-8/60)^10
          .7609322839
1-binompdf(10,8/
60,0)
          .7609322839
```

21 --- De veronderstelling dat voor elke persoon de kans dat hij een kopieerapparaat nodig heeft gelijk is.
--- De veronderstelling dat op elk moment evenveel behoefte is om te kopiëren.

22 K, het aantal klanten dat op dat moment geholpen moet worden, is binomiaal verdeeld met $n = 120$ en $p = \frac{4}{180}$.
$P(K > 3) = P(K \geq 4) = 1 - P(K \leq 3) = 1 - \text{binomcdf}(120, \frac{4}{180}, 3) \approx 0{,}278$.

```
1-binomcdf(120,4
/180,3)
        .2777354996
```

23a F, het aantal drukfouten dat op een willekeurige bladzijde staat, is binomiaal verdeeld met $n = 48$ en $p = \frac{1}{280}$.
$P(F \geq 2) = 1 - P(F \leq 1) = 1 - \text{binomcdf}(48, \frac{1}{280}, 1) \approx 0{,}013$.

```
1-binomcdf(48,1/
280,1)
         .012903068
```

23b $P(F = 2) = \text{binompdf}(48, \frac{1}{280}, 2) \approx 0{,}012$.
Dus je verwacht $\text{Ans} \cdot 280 \approx 3$ bladzijden met twee drukfouten.

```
binompdf(48,1/28
0,2)
        .0122044144
Ans*280
        3.417236033
```

24 J, het aantal jarige docenten op deze dag, is binomiaal verdeeld met $n = 120$ en $p = \frac{1}{365}$.
$P(J \geq 2) = 1 - P(J \leq 1) = 1 - \text{binomcdf}(120, \frac{1}{365}, 1) \approx 0{,}043$.

```
1-binomcdf(120,1
/365,1)
        .0433209115
```

25a $\text{normalcdf}(100, 10^{99}, 80, 12) \approx 0{,}048$.

```
normalcdf(100,10
^99,80,12)
        .0477903304
```

25b $\text{normalcdf}(-10^{99}, a, 80, 12) = 0{,}35$ (intersect of)
$a = \text{invNorm}(0.35, 80, 12) \approx 75{,}38$.

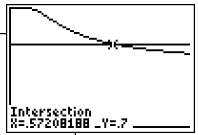
25c $\text{normalcdf}(b, 10^{99}, 80, 12) = 0{,}08$ (intersect of)
$b = \text{invNorm}(0.92, 80, 12) \approx 96{,}86$.

```
invNorm(0.35,80,
12)
        75.37615433
invNorm(1-0.08,8
0,12)
        96.86085873
```

25d $\text{normalcdf}(-10^{99}, 2.1, 1.8, \sigma) = 0.7$
intersect geeft $\sigma \approx 0{,}57$.

```
Plot1 Plot2 Plot3
\Y1=normalcdf(-1
0^99,2.1,1.8,X)
\Y2=0.7
\Y3=
```

```
WINDOW
 Xmin=0
 Xmax=1
 Xscl=0
 Ymin=0
 Ymax=1
 Yscl=0
 Xres=1
```



26a $\text{normalcdf}(1000, 10^{99}, 1005, 6) \approx 0{,}798$. Dus 79,8%.

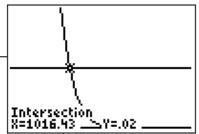
26b $1 - \text{normalcdf}(1001, 1009, 1005, 6) \approx 0{,}505$. Dus 50,5%.

```
normalcdf(1000,1
0^99,1005,6)
        .7976716754
1-normalcdf(1001
,1009,1005,6)
        .504984934
```

26c $\text{normalcdf}(-10^{99}, 1000, \mu, 8) = 0{,}02$ (intersect) $\Rightarrow \mu \approx 1016{,}4$ (gram).

```
Plot1 Plot2 Plot3
\Y1=normalcdf(-1
0^99,1000,X,8)
\Y2=0.02
\Y3=
```

```
WINDOW
 Xmin=1000
 Xmax=1050
 Xscl=0
 Ymin=0
 Ymax=2*0.02
 Yscl=0
 Xres=1
```



27a $\text{normalcdf}(60, 70, 65.8, 9.2) \approx 0{,}412$. Dus 41,2%.

```
normalcdf(60,70,
65.8,9.2)
        .4117875449
```

27b $\text{normalcdf}(-10^{99}, b, 65.8, 9.2) = 0{,}15$ (intersect of)
$b = \text{invNorm}(0.15, 65.8, 9.2) \approx 56{,}3$.
Dus tot de score 56,3 val je af.

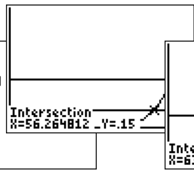
```
invNorm(0.15,65.
8,9.2)
        56.26481291
```

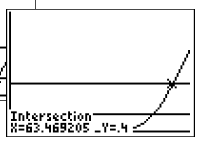
27c $\text{normalcdf}(-10^{99}, c, 65.8, 9.2) = 0{,}40$ (intersect of)
$c = \text{invNorm}(0.40, 65.8, 9.2) \approx 63{,}5$.
Dus bij de scores van 56,3 tot 63,5 mag je herkansen.

```
invNorm(0.40,65.
8,9.2)
        63.46920667
```

```
Plot1 Plot2 Plot3
\Y1=normalcdf(-1
0^99,X,65.8,9.2)
\Y2=0.15
\Y3=0.40
\Y4=
```

```
WINDOW
 Xmin=0
 Xmax=70
 Xscl=0
 Ymin=0
 Ymax=1
 Yscl=0
 Xres=1
```

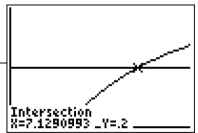




28a $\mu = \frac{28 + 40}{2} = \frac{68}{2} = 34$ (kg). (een normale verdeling is symmetrisch t.o.v. μ)
$\text{normalcdf}(-10^{99}, 28, 34, \sigma) = 0{,}20$ (intersect) $\Rightarrow \sigma \approx 7{,}13$ (kg).

```
Plot1 Plot2 Plot3
\Y1=normalcdf(-1
0^99,28,34,X)
\Y2=0.20
\Y3=
```

```
WINDOW
 Xmin=0
 Xmax=10
 Xscl=0
 Ymin=0
 Ymax=2*0.2
 Yscl=0
 Xres=1
```



28b $\text{normalcdf}(42.8, 10^{99}, 34, 7.13) \approx 0{,}109$. Dus 10,9%.
(als $\sigma \approx 7{,}13$ niet gevonden is, neem dan $\sigma = 7{,}1$)

```
normalcdf(42.8,1
0^99,34,7.13)
        .1085602351
```

28c $\text{normalcdf}(-10^{99}, c, 34, 7.13) = 0{,}05$ (intersect of)
$c = \text{invNorm}(0.05, 34, 7.13) \approx 22{,}3$.
Dus tot 22,3 kg word je opgeroepen.

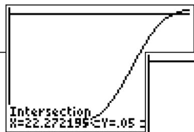
```
invNorm(0.05,34,
7.13)
        22.27219365
```

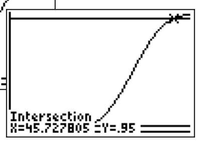
28d $\text{normalcdf}(-10^{99}, d, 34, 7.13) = 0{,}95$ (intersect of)
$d = \text{invNorm}(0.95, 34, 7.13) \approx 45{,}7$. Dus $P_{95} = 45{,}7$ kg.

```
invNorm(0.95,34,
7.13)
        45.72780635
```

```
Plot1 Plot2 Plot3
\Y1=normalcdf(-1
0^99,X,34,7.13)
\Y2=0.05
\Y3=0.95
\Y4=
```

```
WINDOW
 Xmin=0
 Xmax=50
 Xscl=0
 Ymin=0
 Ymax=1
 Yscl=0
 Xres=1
```





29a $P(\text{kind zwaarder dan 42 kg}) = \text{normalcdf}(42, 10^{99}, 34, 7.13) \approx 0{,}131$.

29b $P(\text{minstens één zwaarder dan 42 kg}) = 1 - P(\text{niemand zwaarder dan 42 kg}) = 1 - (1 - \text{Ans})^{10} \approx 0{,}754$.

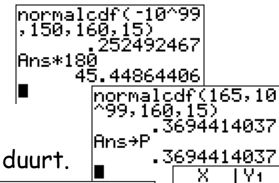
```
normalcdf(42,10^
99,34,7.13)
        .1309270921
1-Ans
        .8690729079
1-Ans^10
        .7542111847
```

29c $P(\text{kind lichter dan 32 kg}) = \text{normalcdf}(-10^{99}, 32, 34, 7.13) \approx 0{,}389...$
C, het aantal kinderen lichter dan 32 kg, is binomiaal verdeeld met $n = 10$ en $p = 0{,}389...$
$P(C \geq 6) = 1 - P(C \leq 5) = 1 - \text{binomcdf}(10, 0.389..., 5) \approx 0{,}149$.

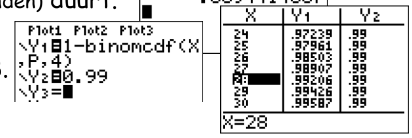
```
normalcdf(-10^99
,32,34,7.13)
        .3895451412
1-binomcdf(10,An
s,5)
        .1492919049
```

30a A is het aantal instellingen dat langer dan 180 seconden (= 3 minuten) duurt.
$p = \text{normalcdf}(180, 10^{99}, 160, 15) = 0{,}091...$ (2 minuten en 40 seconden = 160 seconden)
$P(A \geq 10) = 1 - P(A \leq 9) = 1 - \text{binomcdf}(80, p, 9) \approx 0{,}192$.

```
normalcdf(180,10
^99,160,15)
        .0912112819
1-binomcdf(80,An
s,9)
        .1916743537
```



30b B is het aantal instellingen dat minder dan 150 seconden ($= 2\frac{1}{2}$ minuut) duurt.
$p = \text{normalcdf}(-10^{99}, 150, 160, 15) = 0{,}252...$
Dus naar verwachting duren $p \cdot 180 \approx 45$ handelingen minder dan $2\frac{1}{2}$ minuut.

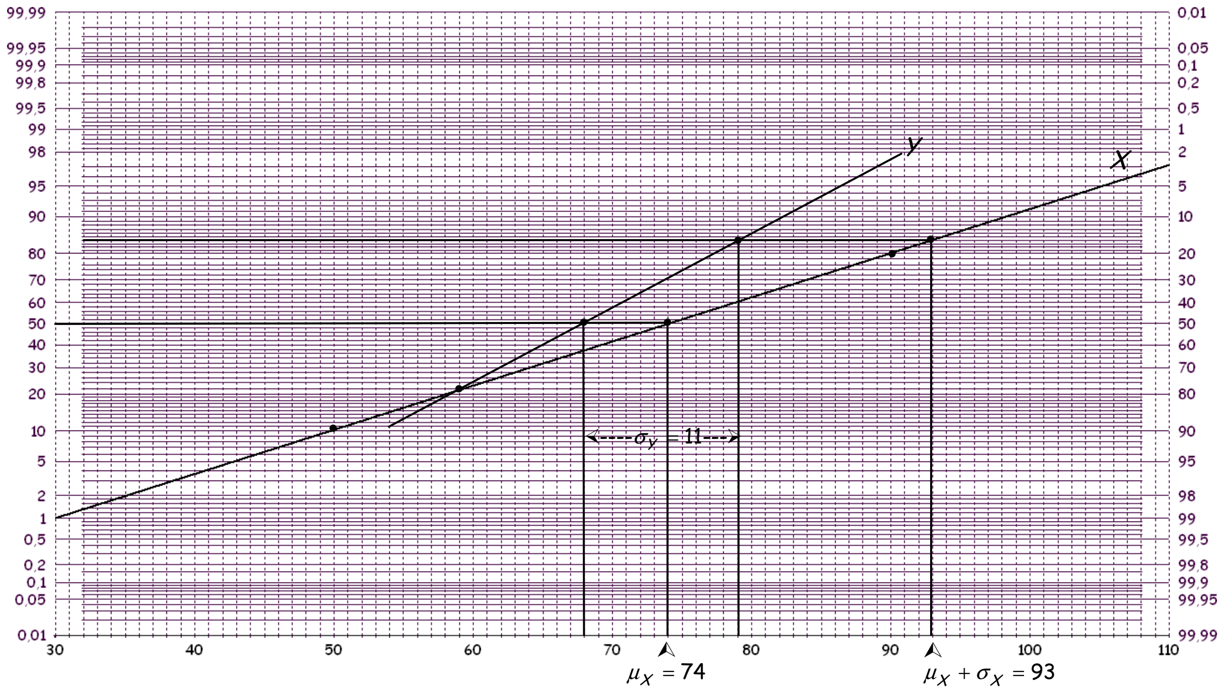


30c C is het aantal instellingen dat meer dan 165 seconden ($=$ 2 minuten en 45 seconden) duurt.
$p = \text{normalcdf}(165, 10^{99}, 160, 15) = 0{,}369...$
$P(C \geq 5) = 1 - P(C \leq 4) = 1 - \text{binomcdf}(n, p, 4) > 0{,}99$ (n geheel $\Rightarrow$ TABLE) $\Rightarrow n \geq 28$.
De werknemer moet minstens 28 remmen instellen.

31a Vuistregel I: 68% van de waarnemingsgetallen ligt binnen één standaardafwijking van $\mu \Rightarrow a = 68$.
Vuistregel II: 95% van de waarnemingsgetallen ligt binnen twee standaardafwijkingen van $\mu \Rightarrow b = 95$.

31b $\mu = \frac{58+67}{2} = \frac{125}{2} = 62{,}5$. (een normale verdeling is symmetrisch t.o.v. μ)
Tussen $P_{2,5}$ en $P_{97,5}$ ligt 95% $\Rightarrow \mu - 2\sigma = P_{2,5} \Rightarrow 62{,}5 - 2\sigma = 58 \Rightarrow -2\sigma = -4{,}5 \Rightarrow \sigma = 2{,}25$.

32a



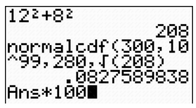
$P_{10} = 50$ en $P_{80} = 90 \Rightarrow$ de lijn van de toevalsvariabel X gaat door (50, 10) en (90, 80) (zie hierboven).
Lees nu af: bij 50% hoort $\mu_X = 74$ en bij 84% hoort $\mu_X + \sigma_X = 93 \Rightarrow \mu_X = 74$ en $\sigma_X = 93 - 74 = 19$.

32b $\mu_Y = 68$ (niet 60 zoals in de eerste boeken) en $\sigma_Y = 11$ (niet 20 zoals in de eerste boeken).
Dus de lijn van de toevalsvariabel Y gaat door (68, 50) en (79, 84) (zie de figuur bij 32a hierboven).

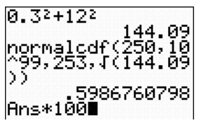
32c Het snijpunt (59, 21) betekent dat van beide toevalsvariabelen 21% van de waarnemingen onder de 59 ligt.

33a Ja, zie figuur 13.9. (de gemiddeldes van beide klokken zijn elkaars tegengestelden $\Rightarrow \mu_{-X} = -\mu_X$)

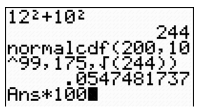
33b Nee, zie figuur 13.9. (de standaardafwijkingen van beide klokken zijn gelijk $\Rightarrow \sigma_{-X} = \sigma_X$)



34 De totale afhandelingstijd $T = X + Y$ van de twee fasen is normaal verdeeld met:
$\mu_T = \mu_X + \mu_Y = 170 + 110 = 280$ (seconden) en $\sigma_T = \sqrt{\sigma_X^2 + \sigma_Y^2} = \sqrt{12^2 + 8^2} = \sqrt{208}$ (seconden).
$P(T > 5 \cdot 60) = \text{normalcdf}(300, 10^{99}, 280, \sqrt{208}) \approx 0{,}083$. Dus in 8,3% van de gevallen.

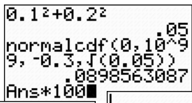


35 De brutogewicht $B = X + Y$ is normaal verdeeld met:
$\mu_B = \mu_X + \mu_Y = 5 + 248 = 253$ (gram) en $\sigma_B = \sqrt{\sigma_X^2 + \sigma_Y^2} = \sqrt{0{,}3^2 + 12^2} = \sqrt{144{,}09}$ (gram).
$P(B > 250) = \text{normalcdf}(250, 10^{99}, 253, \sqrt{144.09}) \approx 0{,}599$. Dus in 59,9% van de gevallen.

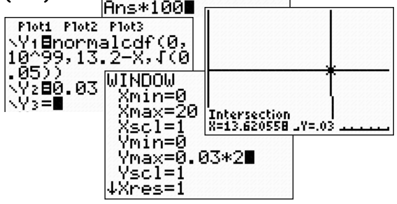


36 De totale afstand $d = d_1 + d_2$ is normaal verdeeld met:
$\mu_d = \mu_{d_1} + \mu_{d_2} = 45 + 130 = 175$ (m) en $\sigma_d = \sqrt{\sigma_{d_1}^2 + \sigma_{d_2}^2} = \sqrt{12^2 + 10^2} = \sqrt{244}$ (m).
$P(T > 200) = \text{normalcdf}(200, 10^{99}, 175, \sqrt{244}) \approx 0{,}055$.

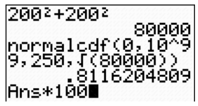
37a De bout is te dik voor de moer als $X > Y \Rightarrow V = X - Y > 0$. V is ook normaal verdeeld met:
$\mu_V = \mu_X - \mu_Y = 13{,}2 - 13{,}5 = -0{,}3$ (mm) en $\sigma_V = \sqrt{\sigma_X^2 + \sigma_Y^2} = \sqrt{0{,}1^2 + 0{,}2^2} = \sqrt{0{,}05}$ (mm).
$P(V > 0) = \text{normalcdf}(0, 10^{99}, -0.3, \sqrt{0.05}) \approx 0{,}090$. Dus in 9,0% van de gevallen.



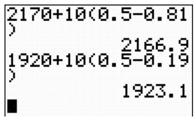
37b $\mu_V = \mu_X - \mu_Y = 13{,}2 - \mu_Y$ (mm) en $\sigma_V = \sqrt{0{,}05}$ (mm).
$P(V > 0) = \text{normalcdf}(0, 10^{99}, 13.2 - \mu_Y, \sqrt{0.05}) = 0{,}03$ (intersect) $\Rightarrow \mu_Y \approx 13{,}62$.
Dus met een gemiddelde diameter van de moeren van 13,62 mm.



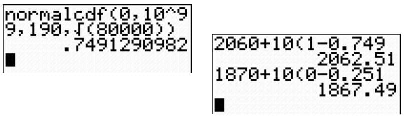
38a A = de speelsterkte van Van der Avoird en T = de speelsterkte van Thijssen.
Van de Avoird wint van Thijssen als $A > T \Rightarrow V = A - T > 0$. V is ook normaal verdeeld met:
$\mu_V = \mu_A - \mu_T = 2170 - 1920 = 250$ en $\sigma_V = \sqrt{\sigma_A^2 + \sigma_T^2} = \sqrt{200^2 + 200^2} = \sqrt{80\,000}$.
$P(V > 0) = \text{normalcdf}(0, 10^{99}, 250, \sqrt{80\,000}) \approx 0{,}812$.



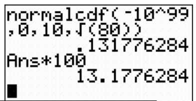
38b De Elo-rating van Van de Avoird: $R_{\text{nieuw}} = R_{\text{oud}} + 10(w - v) = 2170 + 10(0{,}5 - 0{,}81) \approx 2167$.
De Elo-rating van Thijssen: $R_{\text{nieuw}} = R_{\text{oud}} + 10(w - v) = 1920 + 10(0{,}5 - 0{,}19) \approx 1923$.



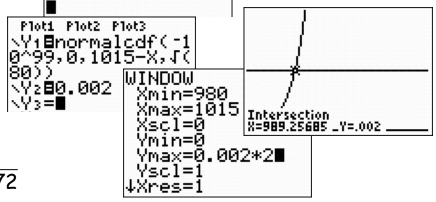
38c K = de speelsterkte van De Keizer en M = de speelsterkte van Mol.
De Keizer wint van Mol als $K > M \Rightarrow V = K - M > 0$. V is ook normaal verdeeld met:
$\mu_V = \mu_K - \mu_M = 2060 - 1870 = 190$ en $\sigma_V = \sqrt{\sigma_K^2 + \sigma_M^2} = \sqrt{200^2 + 200^2} = \sqrt{80\,000}$.
$P(K \text{ wint}) = P(V > 0) = \text{normalcdf}(0, 10^{99}, 190, \sqrt{80\,000}) \approx 0{,}749$.
De Elo-rating van De Keizer wordt $2060 + 10(1 - 0{,}749) \approx 2063$.
De Elo-rating van Thijssen wordt $1870 + 10(0 - 0{,}251) \approx 1867$.



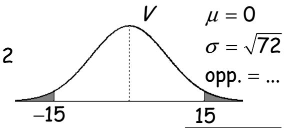
39a Limonade verloren als $X < Y \Rightarrow V = X - Y < 0$. V is ook normaal verdeeld met:
$\mu_V = \mu_X - \mu_Y = 1015 - 1005 = 10$ (ml) en $\sigma_V = \sqrt{\sigma_X^2 + \sigma_Y^2} = \sqrt{4^2 + 8^2} = \sqrt{80}$ (ml).
$P(V < 0) = \text{normalcdf}(-10^{99}, 0, 10, \sqrt{80}) \approx 0{,}132$. Dus in 13,2% van de gevallen.



39b $\mu_V = \mu_X - \mu_Y = 1015 - \mu_Y$ (ml) en $\sigma_V = \sqrt{80}$ (ml).
$P(V < 0) = \text{normalcdf}(-10^{99}, 0, 1015 - \mu_Y, \sqrt{80}) = 0{,}002$ (intersect) $\Rightarrow \mu_Y \approx 989{,}3$.
Dus de machine afstellen op een gemiddelde van 989,3 ml.

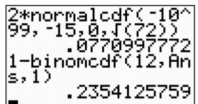


40a X = de lengte van man 1 in cm en Y = de lengte van man 2
Het verschil is meer dan 15 betekent:
$V = X - Y < -15$ (dus $Y - X > 15$) of $V = X - Y > 15$.

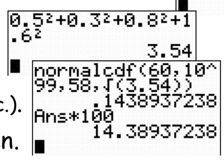


V is normaal verdeeld met: $\mu_V = \mu_X - \mu_Y = 178 - 178 = 0$ (cm) en $\sigma_V = \sqrt{\sigma_X^2 + \sigma_Y^2} = \sqrt{6^2 + 6^2} = \sqrt{72}$ (cm).
opp. $= 2 \cdot \text{normalcdf}(-10^{99}, -15, 0, \sqrt{72}) \approx 0{,}077$ (de gevraagde kans).

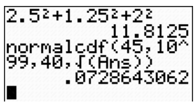
40b T, het aantal tweetallen dat meer dan 15 cm verschilt, is binomiaal verdeeld ($n = 12$ en p = Ans).
$P(T \geq 2) = 1 - P(T \leq 1) = 1 - \text{binomcdf}(12, \text{Ans}, 1) \approx 0{,}235$.



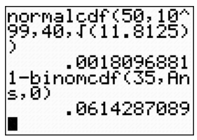
41 De totale afhandelingstijd T is normaal verdeeld met:
$\mu_T = 12 + 8 + 20 + 18 = 58$ (sec.) en $\sigma_T = \sqrt{0{,}5^2 + 0{,}3^2 + 0{,}8^2 + 1{,}6^2} = \sqrt{3{,}54}$ (sec.).
$P(T > 60) = \text{normalcdf}(60, 10^{99}, 58, \sqrt{3.54}) \approx 0{,}144$. Dus in 14,4% van de gevallen.



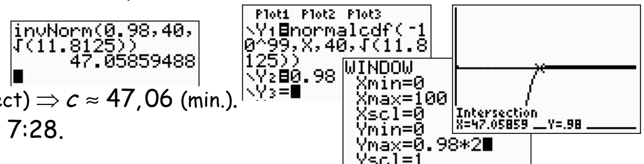
42a De totale fietstijd T is normaal verdeeld met:
$\mu_T = 18 + 7 + 15 = 40$ (min.) en $\sigma_T = \sqrt{2{,}5^2 + 1{,}25^2 + 2^2} = \sqrt{11{,}8125}$ (min.).
$P(T > 45) = \text{normalcdf}(45, 10^{99}, 40, \sqrt{11.8125}) \approx 0{,}073$.



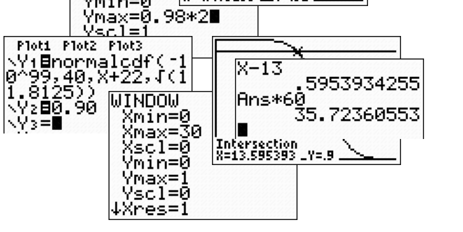
42b Van 7:25 tot 8:15 zijn $35 + 15 = 50$ minuten.
T, het aantal keer te laat, is binomiaal verdeeld met $n = 35$ en $p = \text{normalcdf}(50, 10^{99}, 40, \sqrt{11.8125}) \approx 0{,}0018....$
$P(T \geq 1) = 1 - P(T \leq 0) = 1 - \text{binomcdf}(35, p, 0) \approx 0{,}061$.



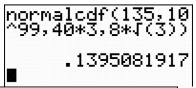
42c $c = \text{invNorm}(0.98, 40, \sqrt{11.8125}) \approx 47{,}06$ (min.).
OF...$\text{normalcdf}(-10^{99}, c, 40, \sqrt{11.8125}) = 0{,}98$ (intersect) $\Rightarrow c \approx 47{,}06$ (min.).
Dus minstens $47 (= 15 + 32)$ minuten vóór 8:15 $\Rightarrow$ vóór 7:28.



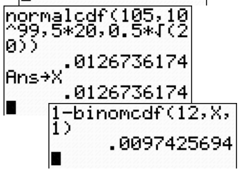
42d $\mu_T = \mu_{\text{I}} + 22$ (min.) en $\sigma_T = \sqrt{11{,}8125}$ (min.).
$\text{normalcdf}(-10^{99}, 40, \mu_{\text{I}} + 22, \sqrt{11.8125}) = 0{,}90$ (intersect) $\Rightarrow \mu_{\text{I}} \approx 13{,}60$ (min.).
Dus de gemiddelde fietstijd op I is (ongeveer) 13 minuten en 36 seconden.



43 $\mu_{\text{som}} = \mu_X + \mu_X + \mu_X + \mu_X + \mu_X + \mu_X + \mu_X + \mu_X = 8 \cdot \mu_X$.
$\sigma_{\text{som}} = \sqrt{\sigma_X^2 + \sigma_X^2 + \sigma_X^2 + \sigma_X^2 + \sigma_X^2 + \sigma_X^2 + \sigma_X^2 + \sigma_X^2} = \sqrt{8 \cdot \sigma_X^2} = \sigma_X \cdot \sqrt{8} = \sqrt{8} \cdot \sigma_X \neq 8 \cdot \sigma_X$.

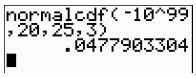


44 $P(X_{\text{som}} \geq 2 \cdot 60 + 1 \cdot 15) = P(X_{\text{som}} \geq 135) = P(X_{\text{som}} > 135) = \text{normalcdf}(135, 10^{99}, 40 \cdot 3{,}8 \cdot \sqrt{3}) \approx 0{,}140$.

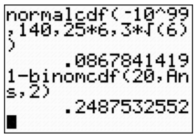


45a $P(X_{\text{stapel}} > 105) = \text{normalcdf}(105, 10^{99}, 5 \cdot 20, 0{,}5 \cdot \sqrt{20}) \approx 0{,}013$.

45b B, het aantal stapels dat niet in de krat past, is binomiaal verdeeld met $n = 12$ en $p = \text{Ans}$.
$P(B \geq 2) = 1 - P(B \leq 1) = 1 - \text{binomcdf}(12, \text{Ans}, 1) \approx 0{,}010$.



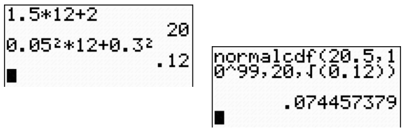
46a $P(X < 20) = \text{normalcdf}(-10^{99}, 20, 25, 3) \approx 0{,}048$.



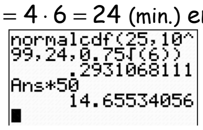
46b $P(B < 140) = \text{normalcdf}(-10^{99}, 140, 25 \cdot 6, 3 \cdot \sqrt{6}) \approx 0{,}087$.

46c C, het aantal pakken dat minder dan 140 gram bevat, is binomiaal verdeeld met $n = 20$ en $p = \text{Ans}$.
$P(C > 2) = P(C \geq 3) = 1 - P(C \leq 2) = 1 - \text{binomcdf}(20, \text{Ans}, 2) \approx 0{,}249$.

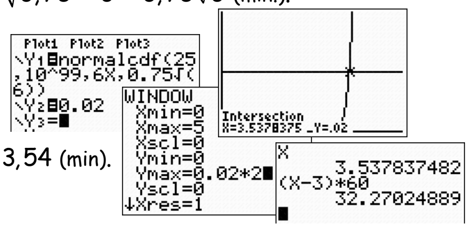
47 T, het gewicht van een krat met 12 flessen, is normaal verdeeld met:
$\mu_T = 1{,}5 \cdot 12 + 2 = 20$ (kg) en $\sigma_T = \sqrt{0{,}05^2 \cdot 12 + 0{,}3^2} = \sqrt{0{,}12}$ (kg).
$P(T > 20{,}5) = \text{normalcdf}(20.5, 10^{99}, 20, \sqrt{0.12}) \approx 0{,}074$.



48a Q, de duur van de quiz, is normaal verdeeld met $\mu_Q = 4 \cdot 6 = 24$ (min.) en $\sigma_Q = \sqrt{0{,}75^2 \cdot 6} = 0{,}75\sqrt{6}$ (min.).
$P(Q > 25) = \text{normalcdf}(25, 10^{99}, 24, 0.75\sqrt{6}) \approx 0{,}293...$
Dus naar verwachting $\text{Ans} \cdot 50 \approx 15$ keer.

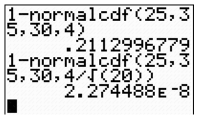


48b De verwachting $P(Q > 25) \cdot 50 \leq 1 \Rightarrow P(Q > 25) \leq \frac{1}{50}$.
$P(Q > 25) = \text{normalcdf}(25, 10^{99}, 6 \cdot \mu_{\text{ronde}}, 0.75\sqrt{6}) = 0{,}02$ (intersect) $\Rightarrow \mu_{\text{ronde}} \approx 3{,}54$ (min).
Dat is 3 minuten en 32 seconden.

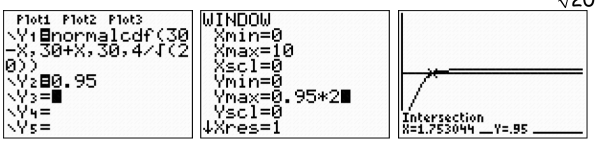


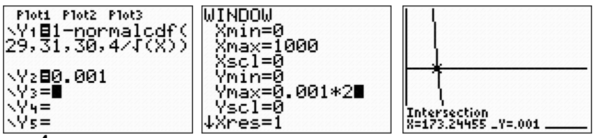
49a $P(X < 25 \vee X > 35) = 1 - P(25 \leq X \leq 35) = 1 - P(25 < X < 35) = 1 - \text{normalcdf}(25, 35, 30, 4) \approx 0{,}211$.

49b $P(\overline{X} < 25 \vee \overline{X} > 35) = 1 - P(25 < \overline{X} < 35) = 1 - \text{normalcdf}(25, 35, 30, \frac{4}{\sqrt{20}}) \approx 0{,}000$.

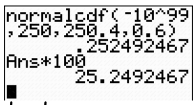


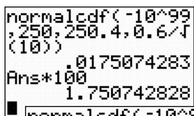
49c $P(30 - a < \overline{X} < 30 + a) = \text{normalcdf}(30 - a, 30 + a, 30, \frac{4}{\sqrt{20}}) = 0{,}95$ (intersect) $\Rightarrow a \approx 1{,}75$.





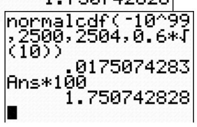
49d $P(\overline{X} < 29 \vee \overline{X} > 31) = 1 - P(29 < \overline{X} < 31) = 1 - \text{normalcdf}(29, 31, 30, \frac{4}{\sqrt{n}}) = 0{,}001$ (intersect) $\Rightarrow n \approx 173{,}2$.
$P(\overline{X} < 29 \vee \overline{X} > 31) < 0{,}001$ (zie een plot/TABLE) $\Rightarrow n > 173$ (of $n \geq 174$).





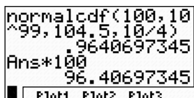
50a $P(A < 250) = \text{normalcdf}(-10^{99}, 250, 250.4, 0.6) \approx 0{,}252$. Dus 25,2% van de pakjes.

50b $P(B < 250) = P(\overline{A} < 250) = \text{normalcdf}(-10^{99}, 250, 250.4, \frac{0.6}{\sqrt{10}}) \approx 0{,}018$. Dus 1,8% van de dozen.

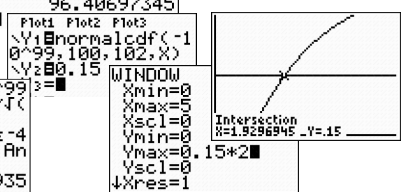


50c $P(C < 2500) = \text{normalcdf}(-10^{99}, 2500, 250.4 \cdot 10, 0.6\sqrt{10}) \approx 0{,}018$. Dus 1,8% van de dozen.

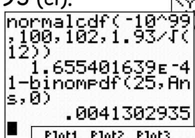
50d Bij een gemiddeld gewicht van 250 gram per pakje, weegt een doos $10 \cdot 250 = 2\,500$ gram.



51 $P(\overline{X} > 100) = \text{normalcdf}(100, 10^{99}, 104.5, \frac{10}{\sqrt{16}}) \approx 0{,}964$. Dus 96,4% van de pakjes.

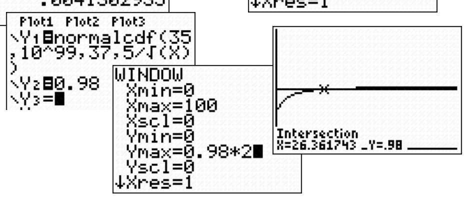


52a $P(A < 100) = \text{normalcdf}(-10^{99}, 100, 102, \sigma) = 0{,}15$ (intersect) $\Rightarrow \sigma \approx 1{,}93$ (cl).



52b $P(B < 100) = P(\overline{A} < 100) = \text{normalcdf}(-10^{99}, 100, 102, \frac{1.93}{\sqrt{12}}) \approx 0{,}0002$.

52c $P(C \geq 1) = 1 - P(C = 0) = 1 - \text{binompdf}(25, \text{Ans}, 0) \approx 0{,}004$.



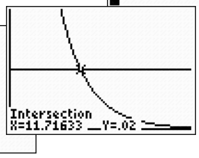
53 $\text{normalcdf}(35, 10^{99}, 37, \frac{5}{\sqrt{n}}) = 0{,}98$ (intersect) $\Rightarrow n \approx 26{,}4$.
$P(\overline{X} < 29 \vee \overline{X} > 31) \geq 0{,}98$ (zie een plot/TABLE) $\Rightarrow n > 26$ (of $n \geq 27$).

54a $P(Z < 5) = \text{normalcdf}(-10^{99}, 5, 5.3, 0.5) \approx 0{,}274.$

54b $P(B < 100) = \text{normalcdf}(-10^{99}, 100, 5.3 \cdot 20, 0.5\sqrt{20}) \approx 0{,}004.$

54c $P(\overline{Z} < 5{,}2 \vee \overline{Z} > 5{,}4) = 1 - P(5{,}2 \le \overline{Z} \le 5{,}4) = 1 - P(5{,}2 < \overline{Z} < 5{,}4 = 1 - \text{normalcdf}(5.2, 5.4, 5.3, \frac{0.5}{\sqrt{20}}) \approx 0{,}371.$
Dus van 37,1% van de pakjes.

54d $P(\overline{Z} < 5) = \text{normalcdf}(-10^{99}, 5, 5.3, \frac{0.5}{\sqrt{n}}) = 0{,}02$ (intersect) $\Rightarrow n \approx 11{,}7.$
$P(\overline{Z} < 5) < 0{,}02$ (zie een plot/TABLE) $\Rightarrow n \ge 12$ (dus minstens 12 zakjes in een pakje).



55a Niet juist, want bij aantallen X is X een geheel getal $\Rightarrow P(X < 4) = P(X \le 3).$

55b Wel juist, want bij gewichten Y is $Y \le 4$ hetzelfde als $Y < 4$.

56a continu. 56c continu. 56e discreet. 56g discreet. 56i discreet.
56b discreet. 56d discreet. 56f discreet. 56h continu. 56j discreet.

57a $P(X \le 10) = P(Y \le 10{,}5).$
57b $P(X < 12) = P(X \le 11) = P(Y \le 11{,}5).$
57c $P(X > 18) = P(X \ge 19) = 1 - P(X \le 18) = 1 - P(Y \le 18{,}5).$
57d $P(X \ge 8) = 1 - P(X \le 7) = 1 - P(Y \le 7{,}5).$
57e $P(6 \le X \le 10) = P(X \le 10) - P(X \le 5) = P(Y \le 10{,}5) - P(Y \le 5{,}5).$
57f $P(8 < X < 20) = P(X \le 19) - P(X \le 8) = P(Y \le 19{,}5) - P(Y \le 8{,}5).$
57g $P(X \le 6 \vee X \ge 8) = P(X \le 6) + P(X \ge 8) = P(X \le 6) + 1 - P(X \le 7) = P(Y \le 6{,}5) + 1 - P(Y \le 7{,}5).$
57h $P(X = 10) = P(X \le 10) - P(X \le 9) = P(Y \le 10{,}5) - P(Y \le 9{,}5).$
57i $P(9 < X \le 15) = P(X \le 15) - P(X \le 9) = P(Y \le 15{,}5) - P(Y \le 9{,}5).$

58a $P(X \le 28) = P(Y \le 28{,}5) = \text{normalcdf}(-10^{99}, 28.5, 35.2, 6.9) \approx 0{,}166.$
58b $P(X \ge 38) = P(Y \ge 37{,}5) = \text{normalcdf}(37.5, 10^{99}, 35.2, 6.9) \approx 0{,}369.$
58c $P(X = 33) = P(32{,}5 \le Y \le 33{,}5) = \text{normalcdf}(32.5, 33.5, 35.2, 6.9) \approx 0{,}055.$
58d $P(30 \le X \le 40) = P(29{,}5 \le Y \le 40{,}5) = \text{normalcdf}(29.5, 40.5, 35.2, 6.9) \approx 0{,}574.$
58e $P(X < 45) = P(Y \le 44{,}5) = \text{normalcdf}(-10^{99}, 44.5, 35.2, 6.9) \approx 0{,}911.$
58f $P(X > 40) = P(Y \ge 40{,}5) = \text{normalcdf}(40.5, 10^{99}, 35.2, 6.9) \approx 0{,}221.$

59a $P(X < 20) = P(Y \le 19{,}5) = \text{normalcdf}(-10^{99}, 19.5, 28.2, 4.3) \approx 0{,}022.$ Dus 2,2%.
59b $P(X = 30) = P(29{,}5 \le Y \le 30{,}5) = \text{normalcdf}(29.5, 30.5, 28.2, 4.3) \approx 0{,}085.$
59c $P(X > 25) = P(Y \ge 25{,}5) = \text{normalcdf}(25.5, 10^{99}, 28.2, 4.3) \approx 0{,}735.$

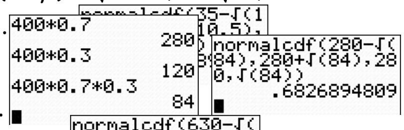
60a $P(X > 12) = P(Y \ge 12{,}5) = \text{normalcdf}(12.5, 10^{99}, 9.8, 3.6) \approx 0{,}227.$
60b $P(X = 10) = P(9{,}5 \le Y \le 10{,}5) = \text{normalcdf}(9.5, 10.5, 9.8, 3.6) \approx 0{,}110.$
60c $P(C \ge 2) = 1 - P(C \le 1) = 1 - \text{binomcdf}(16, 0.2266..., 1) \approx 0{,}907.$

61a $P(X \le 100) = \text{binomcdf}(300, 0.37, 100) \approx 0{,}104.$
61b $P(X \le 100) = P(Y \le 100{,}5) = \text{normalcdf}(-10^{99}, 100.5, 300 \cdot 0.37, \sqrt{300 \cdot 0.37 \cdot (1 - 0.37)}) \approx 0{,}105.$

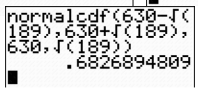
62a X = het aantal personen dat komt opdagen $\Rightarrow P(X \le 1300) = \text{binomcdf}(1430, 0.9, 1300) \approx 0{,}884.$

62b Stel hij accepteert maximaal n reserveringen.
$P(X \le 1300) = \text{binomcdf}(n, 0.9, 1300) > 0{,}99.$
TABLE geeft $n \le 1416$.
Dus hij noteert maximaal 1416 reserveringen.

63 • $n = 50$ en $p = 0{,}7 \Rightarrow np = 50 \cdot 0{,}7 = 35 > 5$ en $n(1-p) = 50 \cdot 0{,}3 = 15 > 5$.
(dus X normaal te benaderen met toevalsvariabele Y waarvan) $\mu_Y = np = 35$ en $\sigma_Y = \sqrt{np(1-p)} = \sqrt{35 \cdot 0{,}3} = \sqrt{10{,}5}$.
$P(35 - \sqrt{10{,}5} < Y < 35 + \sqrt{10{,}5}) = \text{normalcdf}(35 - \sqrt{10.5}, 35 + \sqrt{10.5}, 35, \sqrt{10.5}) \approx 0{,}683$.

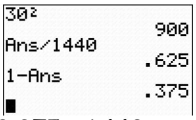


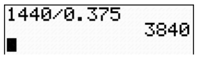
• $n = 400$ en $p = 0{,}7 \Rightarrow \mu_Y = np = 280$ en $\sigma_Y = \sqrt{np(1-p)} = \sqrt{280 \cdot 0{,}3} = \sqrt{84}$.
$P(280 - \sqrt{84} < Y < 280 + \sqrt{84}) = \text{normalcdf}(280 - \sqrt{84}, 280 + \sqrt{84}, 280, \sqrt{84}) \approx 0{,}683$.
• $n = 900$ en $p = 0{,}7 \Rightarrow \mu_Y = np = 630$ en $\sigma_Y = \sqrt{np(1-p)} = \sqrt{630 \cdot 0{,}3} = \sqrt{189}$.
$P(630 - \sqrt{189} < Y < 630 + \sqrt{189}) = \text{normalcdf}(630 - \sqrt{189}, 630 + \sqrt{189}, 630, \sqrt{189}) \approx 0{,}683$.



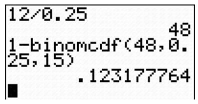
De vuistregel klopt ook bij de binomiaal verdeelde toevalsvariabele X.

64 $\left.\begin{array}{l} E(X) = 1440 \Rightarrow np = 1440 \text{ ❶} \\ \sigma_X = 10 \Rightarrow \sqrt{np(1-p)} = 30 \text{ ❷} \end{array}\right\}$ ❶ in ❷ $\Rightarrow$ $\sqrt{1440(1-p)} = 30$ (kwadrateren)
$1440(1-p) = 900$
$1 - p = \frac{900}{1440}$
$1 - \frac{900}{1440} = p = 0{,}375$ in ❶ $\Rightarrow n \cdot 0{,}375 = 1440 \Rightarrow n = \frac{1440}{0{,}375} = 3840$.



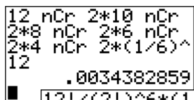


65 $\left.\begin{array}{l} E(X) = 12 \Rightarrow np = 12 \text{ ❶} \\ \sigma_X = 3 \Rightarrow \sqrt{np(1-p)} = 3 \text{ ❷} \end{array}\right\}$ ❶ in ❷ $\Rightarrow$ $\sqrt{12(1-p)} = 3$ (kwadrateren)
$12(1-p) = 9$
$1 - p = \frac{9}{12} = \frac{3}{4}$
$1 - \frac{3}{4} = p = \frac{1}{4}$ in ❶ $\Rightarrow n \cdot \frac{1}{4} = 12 \Rightarrow n = 12 \cdot 4 = 48$.

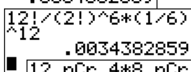


$P(X \geq 16) = 1 - P(X \leq 15) = 1 - \text{binomcdf}(48, \frac{1}{4}, 15) \approx 0{,}1232$.

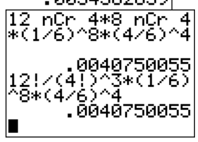
Diagnostische toets



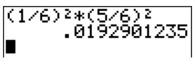
D1a $P(\underline{\underline{112233445566}}) = \binom{12}{2} \cdot \binom{10}{2} \cdot \binom{8}{2} \cdot \binom{6}{2} \cdot \binom{4}{2} \cdot \left(\frac{1}{6}\right)^{12} \approx 0{,}003$ of $\frac{12!}{2! \cdot 2! \cdot 2! \cdot 2! \cdot 2! \cdot 2!} \cdot \left(\frac{1}{6}\right)^{12} \approx 0{,}003$.



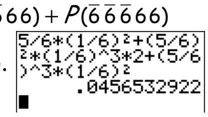
D1b $P(\underline{\underline{11116666aaaa}}) = \binom{12}{4} \cdot \binom{8}{4} \cdot \left(\frac{1}{6}\right)^4 \cdot \left(\frac{1}{6}\right)^4 \cdot \left(\frac{4}{6}\right)^4 \approx 0{,}004$ of $\frac{12!}{4! \cdot 4! \cdot 4!} \cdot \left(\frac{1}{6}\right)^8 \cdot \left(\frac{4}{6}\right)^4 \approx 0{,}004$.



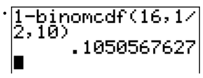
D1c $P(66{*}{*}{*}{*}{*}{*}{*}{*}\bar{6}\bar{6}) = \left(\frac{1}{6}\right)^2 \cdot 1^8 \cdot \left(\frac{5}{6}\right)^2 \approx 0{,}019$. (* mag elk aantal zijn)



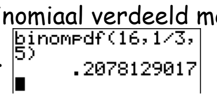
D1d $P(\text{vier of vijf keer gooien}) = P(\text{vier keer gooien}) + P(\text{vijf keer gooien}) = P({*}\bar{6}66) + P(6\bar{6}\bar{6}66) + P(\bar{6}6\bar{6}66) + P(\bar{6}\bar{6}\bar{6}66)$
$= 1 \cdot \frac{5}{6} \cdot \left(\frac{1}{6}\right)^2 + \frac{1}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \left(\frac{1}{6}\right)^2 + \frac{5}{6} \cdot \frac{1}{6} \cdot \frac{5}{6} \cdot \left(\frac{1}{6}\right)^2 + \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \left(\frac{1}{6}\right)^2 \approx 0{,}046$.



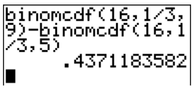
D2a A, het aantal keer even, is binomiaal verdeeld met $n = 16$ en $p = \frac{3}{6} = \frac{1}{2}$.
$P(A > 10) = P(A \geq 11) = 1 - P(A \leq 10) = 1 - \text{binomcdf}(16, \frac{1}{2}, 10) \approx 0{,}105$.



D2b B, het aantal keer 5 of 6 ogen, is binomiaal verdeeld met $n = 16$ en $p = \frac{2}{6} = \frac{1}{3}$.
$P(B = 5) = \text{binompdf}(16, \frac{1}{3}, 5) \approx 0{,}208$.



D2c C, het aantal keer 1 of 2 ogen, is binomiaal verdeeld met $n = 16$ en $p = \frac{2}{6} = \frac{1}{3}$.
$P(5 < C < 10) = P(6 \leq C \leq 9) = \text{binomcdf}(16, \frac{1}{3}, 9) - \text{binomcdf}(16, \frac{1}{3}, 5) \approx 0{,}437$.



vaas	I	II
rood	8	a
wit	$a-8$	$30-a$
totaal	a	30

D3a $P(\underline{\underline{rw}}) = P(rw) + P(wr) = \frac{8}{a} \cdot \frac{30-a}{30} + \frac{a-8}{a} \cdot \frac{a}{30} = \frac{240-8a}{30a} + \frac{a^2-8a}{30a} = \frac{a^2-16a+240}{30a}$.

D3b $P(\underline{\underline{rw}}) = \frac{a^2-16a+240}{30a} = 0{,}6$ (bladeren door TABLE geeft) $a = 10$ of $a = 24$. (a is het aantal knikkers in vaas I)

```
Plot1 Plot2 Plot3
\Y1=(X²-16X+240)/(30X)
\Y2=0.6
\Y3=
```

X	Y1	Y2
8	.73333	.6
9	.65556	.6
10	.6	.6
11	.56061	.6
12	.53333	.6
13	.51538	.6
14	.50476	.6

X=10

X	Y1	Y2
21	.54762	.6
22	.56364	.6
23	.58116	.6
24	.6	.6
25	.62	.6
26	.64103	.6
27	.66296	.6

X=24

D3c $P(\underline{\underline{rw}}) = \binom{2}{1} \cdot P(rw) = 2 \cdot \frac{8}{a} \cdot \frac{a-8}{a-1} = \frac{16(a-8)}{a(a-1)} = \frac{16a-128}{a^2-a}$.

D3d $P(\underline{\underline{rw}}) = \binom{2}{1} \cdot P(rw) = 2 \cdot \frac{a}{30} \cdot \frac{30-a}{29} = \frac{2a(30-a)}{870} = \frac{60a-2a^2}{870}$.

$\frac{2a(30-a)}{870} > 0{,}5$ (bladeren door TABLE) $\Rightarrow a = 13 \vee a = 14 \vee a = 15 \vee a = 16 \vee a = 17$. (rode knikkers in vaas II)

```
Plot1 Plot2 Plot3
\Y1=2X(30-X)/870
\Y2=0.5
\Y3=
```

X	Y1	Y2
12	.49655	.5
13	.50805	.5
14	.51494	.5
15	.51724	.5
16	.51494	.5
17	.50805	.5
18	.49655	.5

X=13

D4a $P(\text{na 2 keer pakken 7 blauwe over}) = P(\underline{rb}) = P(rb) + P(br) = \frac{6}{14} \cdot \frac{8}{14} + \frac{8}{14} \cdot \frac{6}{13} \approx 0{,}509$.

```
6/14*8/14+8/14*6/13
          .5086342229
```

D4b $P(\text{na 3 keer pakken 6 blauwe over}) = P(\underline{rbb}) = P(rbb) + P(brb) + P(bbr) = \frac{6}{14} \cdot \frac{8}{14} \cdot \frac{7}{13} + \frac{8}{14} \cdot \frac{6}{13} \cdot \frac{7}{13} + + \frac{8}{14} \cdot \frac{7}{13} \cdot \frac{6}{12} \approx 0{,}428$.

```
6*8*7/14/14/13+8*6*7/14/13/13+8*7*6/14/13/12
            .42772612
```

D5a $P(\text{Klaas wint in 7 beurten}) = P(\text{Klaas moet na 6 beurten nog 1 punt})$

$= P(\underline{\underline{KKKKKL}}\,K) = P(\underline{\underline{KKKKKL}}) \cdot P(K) = \binom{6}{1} \cdot \left(\frac{2}{6}\right)^5 \cdot \frac{4}{6} \cdot \frac{2}{6} = 6 \cdot \left(\frac{2}{6}\right)^6 \cdot \frac{4}{6} \approx 0{,}005$.

```
6 nCr 1*(2/6)^6*4/6
         .0054869684
```

D5b $P(\text{Leo wint in 7 beurten}) = P(LLLL) + P(\underline{\underline{KLLL}}\,L) = P(LLLL) + P(\underline{\underline{KLLL}}) \cdot P(L) = \left(\frac{4}{6}\right)^4 + \binom{4}{1} \cdot \frac{2}{6} \cdot \left(\frac{4}{6}\right)^3 \cdot \frac{4}{6} \approx 0{,}461$.

```
(4/6)^4+4 nCr 1*2/6*(4/6)^4
         .4609053498
```

D6 K, het aantal klanten dat op dat moment de helpdesk belt, is binomiaal verdeeld met $n = 500$ en $p = \frac{5}{9 \cdot 60} = \frac{5}{540}$.

$P(K > 6) = P(K \geq 7) = 1 - P(K \leq 6) = 1 - \text{binomcdf}(500, \frac{5}{540}, 6) \approx 0{,}185$.

```
1-binomcdf(500,5/540,6)
         .1850543898
```

D7 B, het aantal bouten langer dan 7 mm, is binomiaal verdeeld met $n = 5$ en $p = \text{normalcdf}(7, 10^{99}, 8, 1.3) \approx 0{,}7791...$

$P(B = 5) = p^5 \approx 0{,}287$ (of binompdf$(5, p, 5) \approx 0{,}287$).

```
normalcdf(7,10^99,8,1.3)→X
         .7791219069
X^5
         .2870959584
binompdf(5,X,5)
         .2870959584
```

D8 De totale productietijd T is normaal verdeeld met:

$\mu_T = 19{,}3 + 12{,}5 + 10{,}7 = 42{,}5$ (minc.) en $\sigma_T = \sqrt{2{,}5^2 + 1{,}5^2 + 1{,}2^2} = \sqrt{9{,}94}$ (min.).

$P(T > 45) = \text{normalcdf}(45, 10^{99}, 42.5, \sqrt{9.94}) \approx 0{,}214$. Dus in 21,4% van de gevallen.

```
2.5²+1.5²+1.2²
              9.94
normalcdf(45,10^99,42.5,√(9.94))
          .213902872
Ans*100
```

D9a V, de dikte van de plank na schaven, is normaal verdeeld met:

$\mu_V = 3{,}10 - 0{,}35 = 2{,}75$ (cm) en $\sigma_V = \sqrt{0{,}14^2 + 0{,}09^2} = \sqrt{0{,}0277}$ (cm).

$P(V < 2{,}70) = \text{normalcdf}(-10^{99}, 2.70, 2.75, \sqrt{0.0277}) \approx 0{,}382$. Dus 38,2%.

```
0.14²+0.09²
             .0277
normalcdf(-10^99,2.70,2.75,√(0.0277))
         .3819281321
Ans*100
```

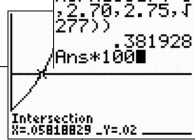
D9b $\mu_V = 3{,}10 - \mu_D$ (cm) en $\sigma_V = \sqrt{0{,}0277}$ (cm).

$P(V < 2{,}70) = \text{normalcdf}(-10^{99}, 2.70, 3.10 - \mu_D, \sqrt{0.0277}) = 0{,}02$ (intersect) $\Rightarrow$

$\mu_D \approx 0{,}06$ (cm). Dus afstellen op een dikte van 0,06 cm (of minder).

```
Plot1 Plot2 Plot3
\Y1=normalcdf(-10^99,2.70,3.10-X,√(0.0277))
\Y2=0.02
\Y3=
```

```
WINDOW
 Xmin=0
 Xmax=.35
 Xscl=0
 Ymin=0
 Ymax=0.02*2
 Yscl=0
↓Xres=1
```



D10a $P(A > 11600) = \text{normalcdf}(11600, 10^{99}, 720 \cdot 16, 14\sqrt{16}) \approx 0{,}077$.

```
normalcdf(11600,10^99,720*16,14*4)
         .0765637714
```

D10b $P(\overline{B} < 710) = \text{normalcdf}(-10^{99}, 710, 720, \frac{14}{\sqrt{16}}) \approx 0{,}002$. Dus 0,2%.

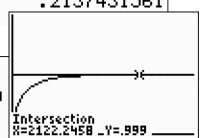
```
normalcdf(-10^99,710,720,14/4)
         .0021374316
Ans*100
         .2137431561
```

D10c $P(719 < \overline{B} < 721) = \text{normalcdf}(719, 721, 720, \frac{14}{\sqrt{n}}) = 0{,}999$ (intersect) $\Rightarrow n \approx 2122{,}2$.

$P(719 < \overline{B} < 721) > 0{,}999$ (zie een plot/TABLE) $\Rightarrow n \geq 2123$ (of $n > 2122$).

```
Plot1 Plot2 Plot3
\Y1=normalcdf(719,721,720,14/√(X))
\Y2=0.999
\Y3=
```

```
WINDOW
 Xmin=0
 Xmax=3000
 Xscl=0
 Ymin=0
 Ymax=2
 Yscl=0
↓Xres=1
```



D11a $P(X = 42 \vee X = 43) = P(41{,}5 \leq Y \leq 43{,}5) = \text{normalcdf}(41.5, 43.5, 42.5, 8.3) \approx 0{,}096$.

D11b $P(X \geq 50) = P(Y \geq 49{,}5) = \text{normalcdf}(49.5, 10^{99}, 42.5, 8.3) \approx 0{,}200$.

```
normalcdf(41.5,43.5,42.5,8.3)
         .0958986942
normalcdf(49.5,10^99,42.5,8.3)
         .1995097422
```

D12a $E(X) = 500 \Rightarrow np = 500$ ❶
$\sigma_X = 20 \Rightarrow \sqrt{np(1-p)} = 20$ ❷

❶ in ❷ $\Rightarrow$ $\sqrt{500(1-p)} = 20$ (kwadrateren)

$500(1-p) = 400$

$1 - p = \frac{400}{500} = \frac{4}{5}$

$1 - \frac{4}{5} = p = \frac{1}{5}$ in ❶ $\Rightarrow n \cdot \frac{1}{5} = 500 \Rightarrow n = 500 \cdot 5 = 2500$.

```
20²
               400
Ans/500
                .8
1-0.8
                .2
```

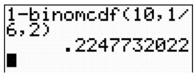
D12b $P(X \geq 525) = 1 - P(X \leq 524) = 1 - \text{binomcdf}(2500, \frac{1}{5}, 524) \approx 0{,}111$.

```
500/0.2
              2500
1-binomcdf(2500,0.2,524)
         .1107404786
```

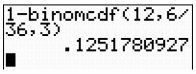
6	7	8	9	10	11	12
5	6	7	8	9	10	11
4	5	6	7	8	9	10
3	4	5	6	7	8	9
2	3	4	5	6	7	8
1	2	3	4	5	6	7
+	1	2	3	4	5	6

Gemengde opgaven 13. Mathematische statistiek

G1a A is het aantal keer 6 ogen bij het werpen met één dobbelsteen.
$P(A > 2) = P(A \geq 3) = 1 - P(A \leq 2) = 1 - \text{binomcdf}(10, \frac{1}{6}, 2) \approx 0{,}225$.



G1b B is het aantal keer meer dan 9 ogen bij het werpen met twee dobbelstenen.
$P(B \geq 4) = 1 - P(X \leq 3) = 1 - \text{binomcdf}(12, \frac{6}{36}, 3) \approx 0{,}125$.

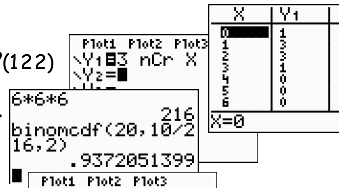


G1c $p = P(\text{som} \leq 5) = P(\text{som} = 3) + P(\text{som} = 4) + P(\text{som} = 5)$

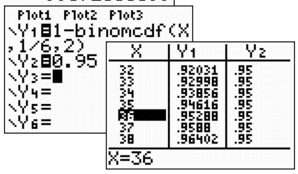
$$= P(111) + P(\underline{112}) + P(\underline{113}) + P(\underline{122}) = P(111) + \binom{3}{2} \cdot P(112) + \binom{3}{2} \cdot P(113) + \binom{3}{1} \cdot P(122)$$

$$= \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} + 3 \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} + 3 \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} + 3 \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{216} + \frac{3}{216} + \frac{3}{216} + \frac{3}{216} = \frac{10}{216}.$$

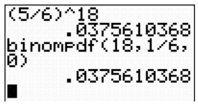
C is het aantal keer hoogstens 5 ogen bij het werpen met drie dobbelstenen.
$P(C \leq 2) = \text{binomcdf}(20, \frac{10}{216}, 2) \approx 0{,}937$.



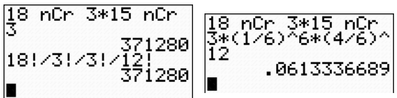
G1d D is het aantal keer 1 oog bij het werpen met één dobbelsteen.
$P(D \geq 3) = 1 - P(D \leq 2) = 1 - \text{binomcdf}(n, \frac{1}{6}, 2) > 0{,}95$ (TABLE) $\Rightarrow n \geq 36$ (of $n > 35$).



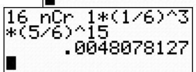
G2a A is het aantal keer 6 ogen bij het werpen met één dobbelsteen.
$P(\text{geen enkele keer } 6) = P(A = 0) = \left(\frac{5}{6}\right)^{18} \approx 0{,}038$. (of binompdf(18, $\frac{1}{6}$, 0))



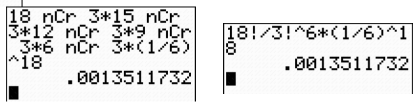
G2b $P(\underline{111666aa...a}) = \binom{18}{3} \cdot \binom{15}{3} \cdot \left(\frac{1}{6}\right)^3 \cdot \left(\frac{1}{6}\right)^3 \cdot \left(\frac{4}{6}\right)^{12} \approx 0{,}061$.
(a staat voor "iets anders dan een 1 of een 6")



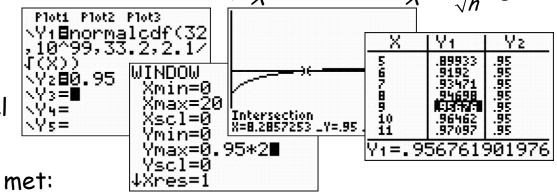
G2c $P(\boxed{666}\,\bar{6}\bar{6}\bar{6}\bar{6}\bar{6}\ldots\bar{6}) = \binom{16}{1} \cdot \left(\frac{1}{6}\right)^3 \cdot \left(\frac{5}{6}\right)^{15} \approx 0{,}005$.
(1 keer "drie 6 naast elkaar" en verder 15 keer "geen 6")



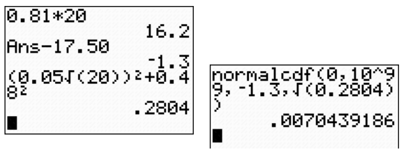
G2d $P(\underline{111222333444555666}) = \binom{18}{3} \cdot \binom{15}{3} \cdot \binom{12}{3} \cdot \binom{9}{3} \cdot \binom{6}{3} \cdot \left(\frac{1}{6}\right)^{18} \approx 0{,}001$.



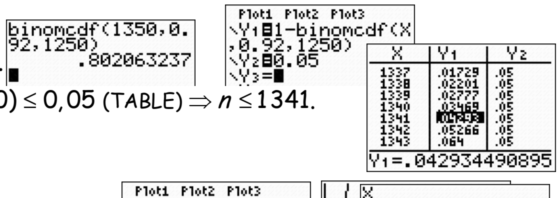
G3 $\overline{X}$, het gemiddelde gewicht van de koeken in een trommel, is normaal verdeeld met $\mu_{\overline{X}} = 33{,}2$ en $\sigma_{\overline{X}} = \frac{2{,}1}{\sqrt{n}}$ (gram).
$P(\overline{X} > 32{,}0) = \text{normalcdf}(32.0, 10^{99}, 33.2, \frac{2.1}{\sqrt{n}}) = 0{,}95 \Rightarrow n \approx 8{,}3$.
$P(\overline{X} > 32{,}0) \geq 0{,}95$ (zie een plot/TABLE) $\Rightarrow n \geq 9$ (of $n > 8$).
Hij doet minstens 9 (of meer dan 8) koeken in de blikken trommel



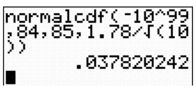
G4 D_{som}, de totale dikte van de 20 tabletten, is normaal verdeeld met:
$\mu_{D_{som}} = 0{,}81 \cdot 20 = 16{,}2$ (cm) en $\sigma_{D_{som}} = 0{,}05 \cdot \sqrt{20}$ (cm).
De tabletten passen niet in het buisje als $D_{som} > L \Rightarrow V = D_{som} - L > 0$.
V is normaal verdeeld met: $\mu_V = \mu_{D_{som}} - \mu_L = 16{,}2 - 17{,}50 = -1{,}3$ (cm)
en $\sigma_V = \sqrt{\sigma^2_{D_{som}} + \sigma^2_L} = \sqrt{0{,}0025 \cdot 20 + 0{,}48^2} = \sqrt{0{,}2804}$ (cm).
$P(V > 0) = \text{normalcdf}(0, 10^{99}, -1.3, \sqrt{0.2804}) \approx 0{,}007$.



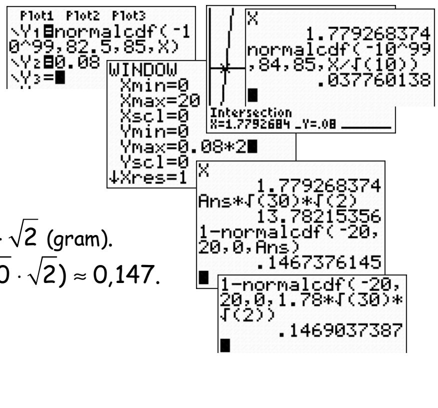
G5a A = het aantal reizen dat niet wordt geannuleerd.
$P(A \leq 1250) = \text{binomcdf}(1350, 0.92, 1250) \approx 0{,}802$ (de gevraagde kans).
G5b $P(A > 1250) = P(A \geq 1251) = 1 - P(A \leq 1250) = 1 - \text{binomcdf}(n, 0.92, 1250) \leq 0{,}05$ (TABLE) $\Rightarrow n \leq 1341$.
Dus men zal maximaal 1341 reizen verkopen.



G6a $P(A < 82{,}5) = \text{normalcdf}(-10^{99}, 82.5, 85, \sigma) = 0{,}08$ (intersect) $\Rightarrow \sigma \approx 1{,}78$ (gram).
G6b $P(\overline{A} < 84) = \text{normalcdf}(-10^{99}, 84, 85, \frac{1.78}{\sqrt{10}}) \approx 0{,}038$.



G6c C, het totale vulgewicht van een doos,
is normaal verdeeld met: $\mu_C = 85 \cdot 30$ (gram) en $\sigma_C = 1{,}78 \cdot \sqrt{30}$ (gram).
V, het verschil in vulgewicht tussen twee dozen, is normaal verdeeld met:
$\mu_V = \mu_C - \mu_C = 0$ (gram) en $\sigma_V = \sqrt{\sigma^2_C + \sigma^2_C} = \sqrt{2 \cdot \sigma^2_C} = \sigma_C \cdot \sqrt{2} = 1{,}78 \cdot \sqrt{30} \cdot \sqrt{2}$ (gram).
$P(\text{verschil minstens } 20) = P(V < -20 \vee V > 20) = 1 - \text{normalcdf}(-20, 20, 0, 1{,}78 \cdot \sqrt{30} \cdot \sqrt{2}) \approx 0{,}147$.



G7a $P(\text{tegoedbon}) = P(\text{kk}) + P(\text{ll}) + P(\text{nn}) + P(\text{vv}) = 0{,}3^2 + 0{,}3^2 + 0{,}3^2 + 0{,}1^2 = 0{,}28.$

```
0.3²*3+0.1²
            .28
```

G7b $P(\text{tegoedbon}) = P(\text{vv}) + P(\text{kk}) + P(\text{ll}) + P(\text{nn}) = k^2 + 3 \cdot (\frac{1}{3} - \frac{1}{3}k)^2 = k^2 + 3 \cdot (\frac{1}{3} - \frac{1}{3}k) \cdot (\frac{1}{3} - \frac{1}{3}k)$

$= k^2 + 3 \cdot (\frac{1}{9} - \frac{1}{9}k - \frac{1}{9}k + \frac{1}{9}k^2) = k^2 + \frac{1}{3} - \frac{1}{3}k - \frac{1}{3}k + \frac{1}{3}k^2 = 1\frac{1}{3}k^2 - \frac{2}{3}k + \frac{1}{3}.$

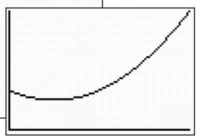
G7c $P(\text{tegoedbon}) = 1\frac{1}{3}k^2 - \frac{2}{3}k + \frac{1}{3} \Rightarrow \frac{dP}{dk} = 2\frac{2}{3}k - \frac{2}{3}.$

$\frac{dP}{dk} = 0 \Rightarrow \frac{8}{3}k - \frac{2}{3} = 0 \ (\times 3) \Rightarrow 8k - 2 = 0 \Rightarrow 8k = 2 \Rightarrow k = \frac{2}{8} = \frac{1}{4}.$

Dus de kans op een tegoedbon is minimaal (volgt uit de vraagstelling of uit een schets) voor $k = \frac{1}{4}$.

```
Plot1 Plot2 Plot3
\Y1=4/3X²-2/3X+1
/3
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```

```
WINDOW
 Xmin=0
 Xmax=1
 Xscl=0
 Ymin=0
 Ymax=1
 Yscl=0
↓Xres=1
```



G7d $P(\text{tegoedbon}) = P(\underline{\underline{\text{vv}\bar{\text{v}}}}) + P(\text{vvv}) = \binom{3}{2} \cdot \left(\frac{1}{4}\right)^2 \cdot \frac{3}{4} + \left(\frac{1}{4}\right)^3 \approx 0{,}156.$

```
3 nCr 2*(1/4)²*3
/4+(1/4)^3
         .15625
```

G8a Tom verwacht $20 \cdot \frac{1}{4} = 5$ juiste antwoorden $\Rightarrow 5 \cdot 1 = 5$ punten.

G8b De verwachtingswaarde van de score *per vraag* is $\frac{1}{4} \cdot 1 + \frac{3}{4} \cdot -0{,}5 = \frac{1}{4} - \frac{3}{8} = \frac{2}{8} - \frac{3}{8} = -\frac{1}{8}$ (punt).

```
1/4*1+3/4*-0.5
          -.125
Ans▸Frac
           -1/8
```

G8c $score = 1 - ({p_A}^2 + (1 - p_B)^2 + {p_C}^2 + {p_D}^2) = 1 - (0{,}2^2 + 0{,}3^2 + 0^2 + 0{,}1^2) = 0{,}86.$

```
1-(0.2²+0.3²+0²+
0.1²)
            .86
```

G8d
- $p_A = 1, \quad p_B = 0, \quad p_C = 0$ en $p_D = 0.$
- $p_A = 0, \quad p_B = 1, \quad p_C = 0$ en $p_D = 0.$
- $p_A = 0, \quad p_B = 0, \quad p_C = 0$ en $p_D = 1.$

G8e *Mogelijkheid II*

Als het juiste antwoord er bij zit (met een kans van $\frac{2}{4}$) is de score $1 - ((\frac{1}{2})^2 + (\frac{1}{2})^2) = \frac{1}{2}.$

Als het juiste antwoord er niet bij zit (met kans van $\frac{2}{4}$) is de score $1 - ((\frac{1}{2})^2 + (\frac{1}{2})^2 + 1^2) = -\frac{1}{2}.$

De verwachte score is dus $\frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot -\frac{1}{2} = \frac{1}{4} - \frac{1}{4} = 0.$

```
1-((1/2)²+(1/2)²
)
             .5
1-((1/2)²+(1/2)²
+1²)
            -.5
```

Mogelijkheid III

Als het juiste antwoord er bij zit (met kans van $\frac{3}{4}$) is de score $1 - ((\frac{1}{3})^2 + (\frac{1}{3})^2 + (\frac{2}{3})^2) = \frac{1}{3}.$

Als het juiste antwoord er niet bij zit (met kans van $\frac{1}{4}$) is de score $1 - ((\frac{1}{3})^2 + (\frac{1}{3})^2 + (\frac{1}{3})^2 + 1^2) = -\frac{1}{3}.$

De verwachte score is dus $\frac{3}{4} \cdot \frac{1}{3} + \frac{1}{4} \cdot -\frac{1}{3} = \frac{3}{12} - \frac{1}{12} = \frac{2}{12} = \frac{1}{6}.$

```
1-((1/3)²+(1/3)²
+(2/3)²)▸Frac
            1/3
1-((1/3)²+(1/3)²
+(1/3)²+1²)▸Frac
           -1/3
```

Conclusie: *Mogelijkheid IV* met een verwachte score van 0,25 (gegeven) is de meest verstandige strategie.

G9a $P(\text{een stuk zeep weegt minder dan 90 gram}) = \text{normalcdf}(-10^{99}, 90, 93, 1.4) \approx 0{,}016.$

$P(\text{alle drie stukken zeep wegen minder dan 90 gram}) = \text{Ans}^3 \approx 0{,}000.$

```
normalcdf(-E99,9
0,93,1.4)
    .0160622279
Ans^3
 4.143977115E-6
0.0161^3
    4.173281E-6
```

G9b $P(T < 460) = \text{normalcdf}(-10^{99}, 460, 93 \cdot 5, 1.4\sqrt{5}) \approx 0{,}055.$

```
normalcdf(-10^99
,460,93*5,1.4√(5
))
    .0551115217
```

G9c $P(\text{een stuk zeep wijkt minder dan } 3\sigma \text{ van } \mu \text{ af}) = \text{normalcdf}(93 - 3 \cdot 1.4, 93 + 3 \cdot 1.4, 93, 1.4) \approx 0{,}997.$

$P(\text{machine opnieuw instellen}) = 1 - P(\text{alle 10 gewichten wijken minder dan } 3\sigma \text{ van } \mu \text{ af}) = 1 - \text{Ans}^{10} \approx 0{,}027.$

```
normalcdf(93-3*1
.4,93+3*1.4,93,1
.4)
    .9973000656
1-Ans^10
    .0266736661
```

G9d $P(\text{een stuk zeep wijkt meer dan } 2\sigma \text{ van } \mu \text{ af}) = 1 - \text{normalcdf}(93 - 2 \cdot 1.4, 93 + 2 \cdot 1.4, 93, 1.4) \approx 0{,}0455.$

(of volgens de tweede vuisteregel een kans van 100% − 95% = 5%)

$P(\text{machine opnieuw instellen}) = 0{,}0455^2 + 0{,}9545 \cdot 0{,}0455^2 \approx 0{,}004.$

(of met gebruik van de tweede vuisteregel $0{,}05^2 + 0{,}95 \cdot 0{,}05^2 \approx 0{,}005$)

```
1-normalcdf(93-2
*1.4,93+2*1.4,93
,1.4)
     .045500124
0.0455²+0.9545*0
.0455²
    .0040463036
```

```
0.05²+0.95*0.05²
        .004875
```

G10a X = de levensduur in uur van de linker koplamp.

$P(X < 2100) = \text{normalcdf}(-10^{99}, 2100, 2500, 450) \approx 0{,}187...$

$P(\text{beide koplampen binnen 2100 branduren stuk}) = \text{Ans}^2 \approx 0{,}035.$

```
normalcdf(-10^99
,2100,2500,450
    .1870313608
Ans²
    .0349807299
0.187²
        .034969
```

G10b Y = de levensduur in uur van de rechter koplamp.

$V = X - Y$ is normaal verdeeld met $\mu_V = \mu_X - \mu_Y = 2500 - 2500 = 0$ (branduren)

en $\sigma_V = \sqrt{\sigma_X^2 + \sigma_Y^2} = \sqrt{2 \cdot \sigma_X^2} = \sigma_X \cdot \sqrt{2} = 450\sqrt{2}$ (branduren).

$P(\text{verschil kleiner dan 20 branduren}) = P(-20 < V < 20) = \text{normalcdf}(-20, 20, 0, 450\sqrt{2}) \approx 0{,}025.$

```
normalcdf(-20,20
,0,450*√(2))
    .0250710763
```